NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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BULLETIN

AN INVESTIGATION OF THE VALIDITY OF WANG'S FORMULA FOR THE CRITICAL LOAD FOR CIRCULAR CYLINDRICAL GRIDS*

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Adapted from a report by T. K. Wang and A. S. Niles
of Stanford University

SUMMARY

Four circular cylindrical grids were tested under axial compression to investigate the validity of a theoretical formula for the critical load for such structures. The chief result of the investigation was to throw light on some difficulties connected with the experimental validation of such formulas.

INTRODUCTION

The modern trend of monocoque and semimonocoque construction in aircraft has placed considerable emphasis on the determination of satisfactory and adequate design criterions.

The subtle interaction of the various parts of a semimonocoque cylinder - namely, the skin, the longitudinal stiffeners and the transverse rings - is of such a nature as to present a very difficult problem for solution either analytically or experimentally.

The problem of general instability of stiffened metal cylinders has been investigated in the Committee's laboratories and at both the California Institute of Technology and the Polytechnic Institute of Brooklyn under the

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*A complete report on this investigation entitled "Compression Tests on Circular Cylindrical Grids," by T. K. Wang and A. S. Niles, is available for reference or loan in the Office of Aeronautical Intelligence, National Advisory Committee for Aeronautics, Washington, D. C.

sponsorship of the Committee. The results of these investigations are at present being organized in order that they may be made available for ready reference to the design engineer.

T. K. Wang, working independently at Stanford University, theoretically developed a formula for the calculation of the critical load of semimonocoque cylinders under axial compression (reference 1). This formula was developed for use with structures composed of continuous smooth skin reinforced by longitudinal stiffeners and transverse rings, like conventional metal airplane fuselages. In the development of the formula it was assumed that the most of the skin buckled under a very low load, and thereafter had no influence on the action of the structures. On this assumption the semimonocoque cylinder became a circular cylindrical grid or "bird cage" type structure.

Considerable interest has been shown regarding this formula, and if it could be shown experimentally that the assumptions used in developing it were valid, it would be a worthy contribution to structural analysis. Realizing the extreme difficulties that might be encountered in the experimental work necessary to confirm Wang's formula and at the same time realizing the great value to the design engineer if the formula could be validated, the Committee sponsored an experimental investigation at Stanford University to attempt to confirm Wang's formula.

In spite of the extreme care and precision that was employed in making the study, it failed either to validate or to invalidate Wang's formula. The interest that has been shown in Wang's theoretical studies has been such that it was considered desirable to bring the results of this investigation to the attention of the structural research engineer. Accordingly, it is the purpose of this report to give only a brief description of the investigation and to present the data that were gathered. As mentioned in the footnote on the first page, the detailed report on this investigation may be obtained by addressing a request to the Office of Aeronautical Intelligence, of the National Advisory Committee for Aeronautics, Washington, D. C.

DESCRIPTION OF TEST SPECIMENS AND APPARATUS

Four specimens were tested in this investigation. Three preliminary specimens of the type shown in figure 1 were made up of 4 steel rings spaced 3 inches apart and 12 round rods, 15 inches in length, equally spaced around the periphery of the rings and lashed to the rings by fishing line tautened by airplane dope. Three sizes of ring material and three sizes of drill rod were used for these specimens. The pertinent dimensions of these specimens are given in table 1. Owing to the difference in size of the rods and rings of these specimens, copper shims were used to maintain a constant diameter for the circle of contact between the rods and the end fittings.

The main test specimen (fig. 2) was constructed of rods of cold rolled steel 15 inches long and 3/8 by 3/16 inch in cross section. The rings of preliminary specimen 2 were not injured in the test and were the ones used in the main specimen. These rings had a mean diameter of 5.32 inches, a radial thickness of 0.1576 inch, and a thickness normal to their midplanes of 0.1945 inch. As in the preliminary specimens, the rings were spaced 3 inches apart, center to center, and the rods were equally spaced with their wider sides tangent to the outside surfaces of the rings. The rods were attached to the rings by machine steel clips, as can be noted in figure 2. The rod ends were machined to leave small hemispherical projections at the ends of their axes where they came in contact with the end fittings.

The stiffnesses, EI, of the rods used in the preliminary specimens were checked by deflection tests, with the rods loaded as simple beams subjected to concentrated loads at midspan. The resulting values of EI are shown in table I. No stiffness tests were run on the rods of the main specimen, the value of I being determined from the nominal dimensions of the rod with E taken as the standard value for steel.

The stiffness of the rings was determined experimentally. This was done by applying a tension load across the diameter of the rings and measuring the increase in diameter. The experimentally determined values of EI for the rings are listed in table 1.

The complete models were tested in compression with a Tinius Olsen 20,000-pound-capacity, hand-operated,

universal-testing machine. Special end fittings (fig. 3) were constructed to permit small changes to be made in the location of the specimen and to obtain more uniform distribution of the load between the rods. Figure 4 shows one of the preliminary specimens mounted in the testing machine. As can be noted, the load was transmitted to the specimen through a ball bearing, into the upper fitting, and subsequently into the rods of the specimen through bearing screws in the upper fitting. Effort was made to maintain a uniform distribution of load in the rods by adjusting these bearing screws.

Four dial gages were mounted around the periphery of the upper fitting as shown in figure 4 to indicate, during the loading of the specimen, any deviation of the upper fitting from a plane parallel to the plane of support of the lower fitting. This was done to indicate any eccentricity of the applied load. The upper fitting was restrained against rotation by means of a pin rigidly fastened to the testing machine which passed through a slot in the upper fitting.

The rings also were restrained against rotation by lashing prong bars to the rings. The prongs fitted in slots in a pillar attached to the lower contact plate and supported at the top by a brace. The pillar and restraining bars are shown in figure 3. The rings supported in this manner were free to deform in the plane of their diameter and to move vertically, being restrained only against rotation, thus the stiffness of the rings was not increased.

As tested in this apparatus, the rods of the cylindrical grid deflected in parallel planes when failure took place. The loads at failure, however, were much lower than those computed from the formulas of reference 1. Although these failures appeared to take place without rotation of the specimen about a vertical axis, in the test of the main specimen dial gages were used to measure tangential movement of one point in the upper ring and one point in the next to the lowest ring.

TEST OF MAIN SPECIMEN

The main specimen was tested with the apparatus and procedure developed as outlined above. The specimen was first centered under a tare load of 16 pounds, due to the

dead weight of the upper fitting. To check the centering operation the load was increased by several thousand pounds, and if the dial gages measuring the vertical movement of the upper contact plate showed appreciable rotation of that member, the load was reduced to the tare value, and the model moved by operating the adjusting screws of the end fittings. After the model had been suitably centered, the dial gages for measuring horizontal movement of the rings were installed and all dial gages set to read zero.

Load was then applied in increments of 2000 pounds until 9000 pounds was reached, and then in increments of 1000 pounds until 18,000 pounds was attained. At each increment of loading the four gages measuring the vertical movement of the lugs on the upper contact plate and the two gages measuring the tangential movement of points on rings were read, and a check reading was made of the imposed load before adding another increment. At 18,000 pounds the check reading for the first time indicated a drop in load while the gage readings were being taken. The dial gages measuring tangential ring movement also were beginning to show excessive readings, and were removed to prevent injury. Three more load increments were imposed, and, after each of them, the load dropped while the remaining gages were being read, as is shown by the test log reproduced in table 2. After the last of these increments the load applied through the testing machine dropped from 19,100 to 16,950 pounds while the gages were being read, and further straining of the specimen failed to produce increased resistance. The test was therefore discontinued.

After completion of this test the specimen was disassembled and the inside diameter of each ring was measured in 6 places, at intervals of 30 degrees. The results of these measurements are listed in table 3, where the data are so tabulated that all values in a single column represent diameters that were coplanar when the model was tested. Before the model was disassembled it was noted that some of the rods appeared to be slightly bent in the tangential direction, but it was not found possible to obtain accurate and consistent measurements of the amount of deformation.

DISCUSSION

The primary value of the tests made in this investigation was to throw additional light on some of the practical difficulties in validating Wang's formula for the critical load of a circular cylinder of semimonocoque construction. The difficulties may be summarized as

1. Correct determination of effective stiffness of rods and rings if true semimonocoque construction with smooth sheet covering is used

2. Prevention of torsional failure without strengthening the structure against the desired type of general instability failure to an indeterminate degree

3. Prevention of failure in which the rods deflect in half waves of length equal to the total length of the specimen instead of the type of general instability failure indicated by theory

The first of these difficulties was circumbented by the use of grids instead of covered cylinders, but at the expense of intensification of the other two. In fact, both of the other two difficulties were due primarily to the lack of a covering to stiffen the rods against tangential bending.

Torsional failure of the preliminary specimen was apparently prevented by the test apparatus but at the expense of decreasing the effectiveness of the provisions made for centering the load.

The success of the provisions for preventing twist in the test of the main specimen is questionable as the rods, or at least some of them, actually bent tangentially when failure occurred. The amount of permanent deformation, however, was so slight that it was difficult to say whether the major action of the rings was rotation, movement in translation, or deformation to an elliptical shape.

According to the theory of reference 1 the rods of the preliminary specimens should have buckled in three, and those of the main specimen in two, half waves; while the rings buckled to an elliptic shape. Actually they seemed to buckle into a single half wave for each rod, and only with the main specimen was there any indication

of elliptical deformation of the rings. The circular rods of the preliminary specimen might be expected to fail in any direction inasmuch as their stiffness with respect to bending was the same in all directions; the only restraint to failure in any direction being offered by the rings and connections.

To eliminate the possibility of tangential deflection of the rods in the main specimen in order to obtain the type of failure envisaged in the theory of reference 1, it would have been desirable to have so proportioned the rods that their Euler loads based on EI_{st} and a length of 15 inches would exceed that based on EI_{sr} and a length of 3 inches. To accomplish this, the ratio of width of rod to depth would have to have been at least 5:1. This did not seem practical; so the rods were proportioned 2:1. This was probably one reason the rods of the main specimen actually bent tangentially when failure took place.

On the other hand, the total Euler load for the twelve rods of the main specimen computed on the basis of EI_{st} and $L = 15$ inches was only about 13,000 pounds; while the specimen did not fail until the imposed total load was in excess of 19,000 pounds. This trend also was true for the preliminary specimens; but the results were very erratic, specimen 2 carrying much more than specimen 1, although it was made with lighter rods and also lighter rings. Thus it hardly can be denied that the rings increased the carrying capacity of the rods, though the test data are too few and too conflicting to be of any practical value as guides in design.

Exception may be taken to the method used to insure uniform bearing of the upper contact plate on the 12 rods. It was admittedly crude, but it is believed that inequalities of bearing had little if any influence on the loads under which the specimens failed.

As noted in table I, the ultimate load of the main specimen was considerably higher than the sum of the Euler loads of the rods considered as separate columns, yet it was considerably below that of equation 13 of reference 1. The fact that the ultimate loads measured did not agree with the loads predicted by theory would tend to invalidate the theory; however the test setup did not cause the grids to fail in accordance with the theory,

and for that reason the tests were inconclusive. On the whole, it cannot be said that the tests either validated or invalidated the theory of reference 1.

The observed permanent set of the rings, however, suggests that there may be merit in the theory of reference 1, which was being investigated, but the evidence was far from conclusive.

CONCLUSIONS

1. Care must be taken to recognize that the formulas of reference 1 apply only to a particular type of instability failure, and that there are other possible modes of failure which may be associated with lower critical loads.

2. As yet, no fully satisfactory method has been developed for the experimental validation of those formulas.

National Advisory Committee for Aeronautics,
Washington, D. C.

REFERENCE

1. Wang, Tsun Kuei: Buckling of Semimonocoque Structures under Compression. Jour. of Applied Mechanics, vol. 9, no. 3, Sept. 1942, pp. A117-A121.

TABLE 1.- SPECIMEN DIMENSIONS AND ULTIMATE LOADS

	Preliminary specimens 1	Preliminary specimens 2	Preliminary specimens 3	Main specimen
Rods, diameter, in.	0.157	0.143	0.120	Note 1
EI_{sr}, lb-in.^{2}	882	614	228	6180
$I_{sr} \times 1000$, in.^{4}	0.0298	0.0207	0.01017	0.206
Total length, in.	15	15	15	15
Distance between rings, in. .	3	3	3	3
Rings, mean radius, R, in. . . .	2.675	2.660	2.650	2.660
Radial thickness, in. . .	0.1854	0.1576	0.1248	0.1576
Normal thickness, in. . .	0.1883	0.1945	0.1875	0.1945
EI_r, lb-in.^{2}	3038	1948	923	1948
$I_r \times 1000$, in.^{4}	0.1000	0.0634	0.0304	0.0634
Computed total instability load, lb (Euler loads)				
L = 3 in.	11,600	8,100	3,000	81,400
L = 15 in.	464	324	120	Note 2
Computed general instability load, lb (Note 4).	10,100	6,730	2,480	29,500
Ultimate load in test, lb . . .	606	1,016	500	19,100

Notes: 1. Rods of main specimen were 3/8 by 3/16 in. in section.

2. Computed unit instability load 3,256 or 13,024 depending on assumed direction of bending.

3. For circular rods $I_{st} = I_{sr}$, for rectangular, $I_{st} = 4\, I_{sr}$.

4. Computed from Wang's formula (equation (13), reference 1).

TABLE 2.- TEST LOG

[Gage readings in 0.001 inch]

Load (lb)	Compression gages				Rotation gages	
	No. 1	No. 2	No. 3	No. 4	No. N	No. S
1,000	0	0	0	0	0	0
3,000	$3\frac{1}{2}$	$3\frac{3}{4}$	$3\frac{1}{2}$	$3\frac{1}{2}$	1	2
5,000	5	$5\frac{1}{2}$	$5\frac{3}{4}$	6	$1\frac{1}{2}$	$8\frac{1}{2}$
7,000	7	7	$8\frac{1}{4}$	8	1	$13\frac{1}{2}$
9,000	$8\frac{1}{2}$	8	$9\frac{1}{4}$	10	-1	$18\frac{1}{2}$
10,000	$9\frac{1}{2}$	9	$10\frac{1}{4}$	$10\frac{1}{2}$	$-1\frac{1}{2}$	$20\frac{1}{2}$
11,000	$10\frac{3}{4}$	10	$11\frac{1}{4}$	$11\frac{1}{4}$	$-1\frac{1}{2}$	21
12,000	$12\frac{3}{4}$	11	12	$12\frac{1}{4}$	$-1\frac{1}{2}$	22
13,000	14	12	$12\frac{3}{4}$	$13\frac{1}{4}$	$-1\frac{1}{2}$	23
14,000	$15\frac{3}{4}$	$13\frac{1}{4}$	$13\frac{1}{4}$	$14\frac{1}{4}$	-2	$24\frac{1}{8}$
15,000	17	$14\frac{3}{4}$	$14\frac{3}{4}$	$15\frac{1}{2}$	-5	28
16,000	$18\frac{1}{2}$	$15\frac{3}{4}$	16	$15\frac{1}{2}$	$-8\frac{1}{2}$	$32\frac{3}{4}$
17,000	20	$17\frac{1}{2}$	$16\frac{3}{4}$	18	-17	$42\frac{1}{2}$
18,000	$21\frac{3}{4}$	19	$18\frac{1}{4}$	19	-35	44
Load dropped to 17,885 while readings were being taken						
18,700	24	$21\frac{1}{4}$	$19\frac{3}{4}$	21		
Load dropped to 18,610 while readings were being taken						
19,085	29	$25\frac{1}{2}$	$22\frac{1}{2}$	25		
Load dropped to 19,060 while readings were being taken						
19,100	47	42	30	35		

Load dropped to 16,950 while readings were being taken and the loading could not increase any more.

TABLE 3.- INSIDE DIAMETERS OF RINGS AFTER TESTING

Ring	Station					
A	5.220	5.214	5.164	5.120	5.129	5.169
B	5.216	5.206	5.155	5.130	5.128	5.167
C	5.199	5.198	5.156	5.132	5.149	5.201
D	5.191	5.186	5.174	5.136	5.146	5.200

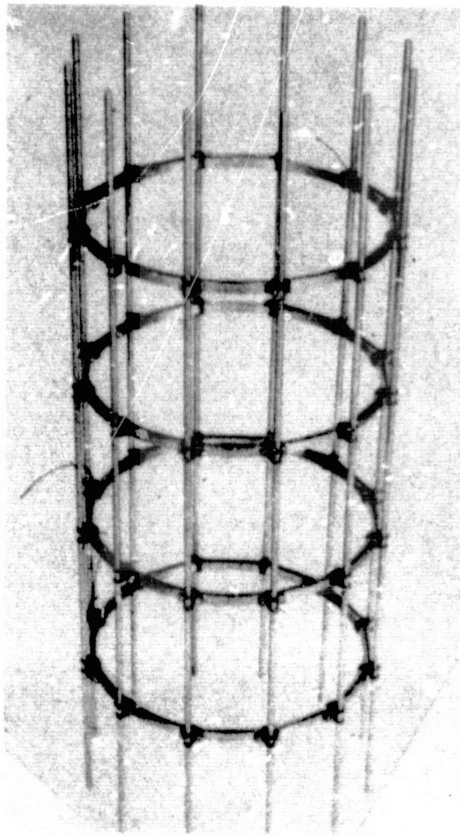


Figure 1.- Preliminary test specimen.

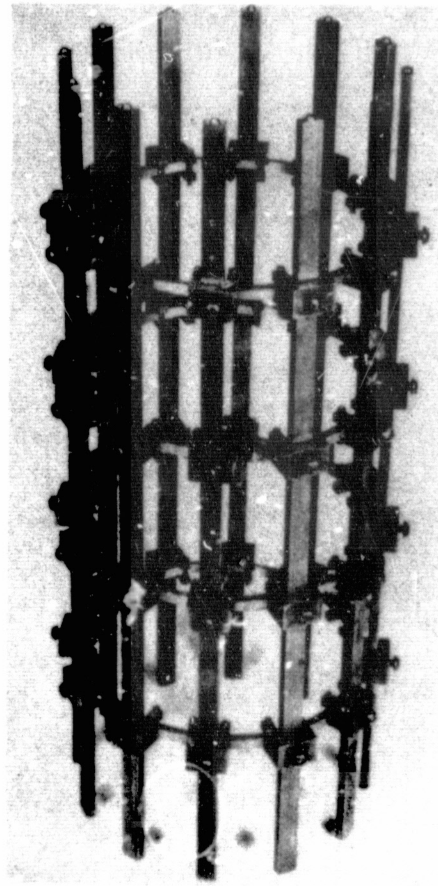


Figure 2.- Main test specimen.

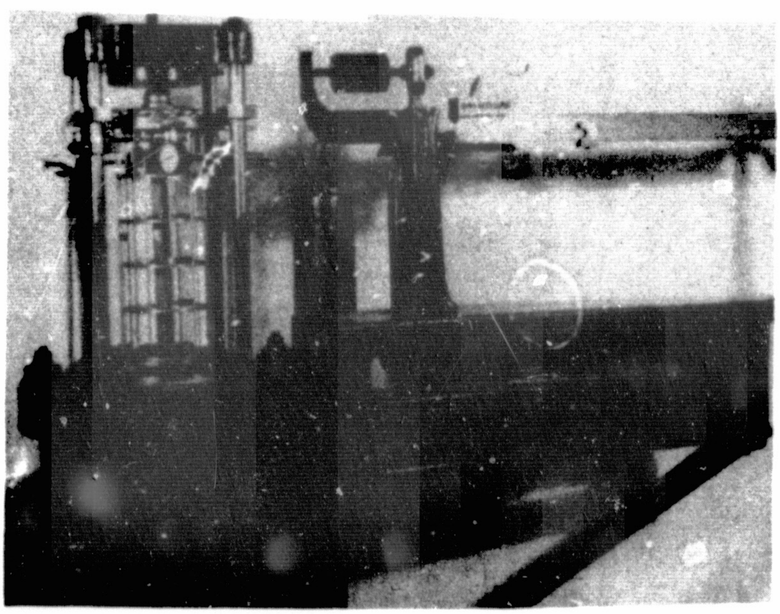
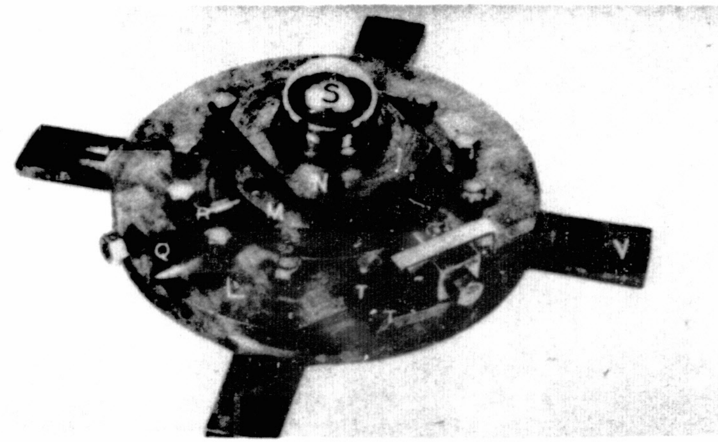
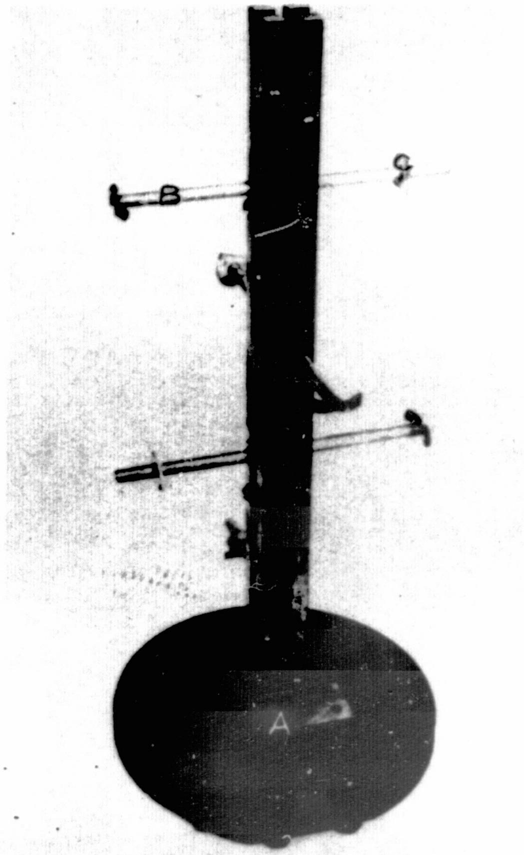


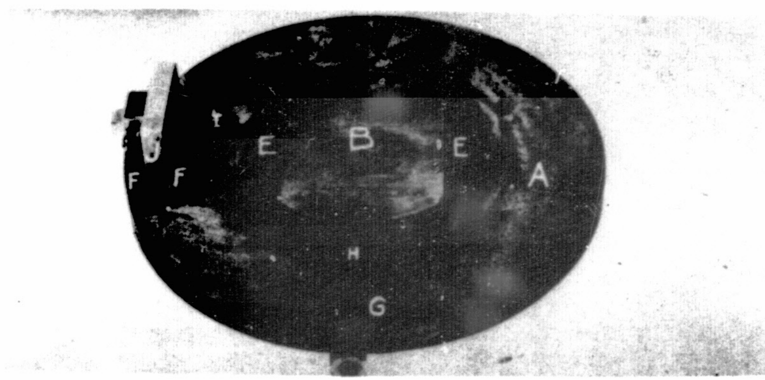
Figure 4.- Preliminary specimen ready for test.



(a) Upper fitting, assembled.



(b) Lower contact plate with central pillar.



(c) Lower base and intermediate plates.

Figure 3.- Special end fittings to permit concentric loading of the specimen and uniform distribution of load in the rods, and the apparatus to prevent rotation of the rings.